Grain-size dependence of shear wave speed dispersion and attenuation in granular marine sediments

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Abstract: The author has shown that measured shear wave speed dispersion and attenuation in water-saturated silica sand can be predicted by using a gap stiffness model incorporated into the Biot model (the BIMGS model) [Kimura, J. Acoust. Soc. Am. 134, 144–155 (2013)]. In this study, the grain-size dependence of shear wave speed dispersion and attenuation in four kinds of water-saturated silica sands with different grain sizes is measured and calculated. As a result, the grain-size dependence of the aspect ratio in the BIMGS model can be validated and the effects of multiple scattering for larger grain sizes are demonstrated.

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1. Introduction

The frequency dependence of the speed and attenuation of compressional waves in granular marine sediments has been widely investigated theoretically and experimentally, while the study on the shear wave speed dispersion and attenuation has been quite insufficient. Brunson reported measurements of the frequency dependence of shear wave attenuation in the frequency range of 0.45–7 kHz for water-saturated sand, and of 1–20 kHz for water-saturated glass beads, sorted sand, and unsorted sand. Even in Brunson’s data, the shear wave speed data are available at only one frequency, i.e., 10 kHz. In addition, the grain sizes used were almost the same. Therefore, the grain-size dependence of the shear wave speed dispersion and attenuation could not be obtained. Recently, the author measured the temperature dependence of shear wave speed dispersion and attenuation in water-saturated silica sand which grain size is 0.113 mm in the frequency range of 4–20 kHz, and showed that the measured results can be predicted by using a gap stiffness model incorporated into the Biot model (the BIMGS model). The author has also derived the relationship between the aspect ratio in the BIMGS model and the grain size $\phi = -\log_2 d$, $d$ in mm), for both compressional and shear waves.

In this study, the grain-size dependence of shear wave speed dispersion and attenuation in four kinds of water-saturated silica sands with different grain sizes are measured. The measured results are compared with the calculated results using the BIMGS model, and the grain-size dependence of the shear wave speed dispersion and attenuation are investigated.

2. The BIMGS model

A modified gap stiffness model incorporated into the Biot model, which is called the BIMGS model, has been developed by the author. This model is an improved version of the Chotiros and Isakson model (the BICSQS model). The acoustic relaxation due to the local fluid flow in the gap between the grains is described in this
model. The effective normal stiffness, \( k_n \), is the sum of the contact stiffness, \( k_c \), and the modified gap stiffness, \( k_g \); this can be expressed as follows:

\[
k_n = k_c + k_g. \tag{1}
\]

The modified gap stiffness, \( k_g \), is expressed as follows:\(^5,6\)

\[
k_g = \frac{\pi a^2 K_f}{h_0} \left\{ 1 - \frac{1}{\kappa a J_0(\kappa a)} \right\}, \tag{2}
\]

where \( a \) denotes the contact radius; \( h_0 \), the initial value of the gap separation distance; \( K_f \), the fluid bulk modulus; \( J_0 \) and \( J_1 \) represent the Bessel functions of the first kind of the zeroth and first orders, respectively, and \( \kappa a \) is defined as

\[
\kappa a = \sqrt{-j2\pi f f_r}, \tag{3}
\]

where \( f \) denotes frequency. \( f_r \) represents the relaxation frequency of the modified gap stiffness that depends on the bulk modulus \( K_f \), the viscosity \( \eta \) of the pore fluid, and the aspect ratio \( \kappa(=h_0/a) \) as follows:

\[
f_r = \frac{1}{12} \frac{K_f}{\eta} \kappa^2. \tag{4}
\]

The frame shear modulus, \( \mu_b \), is determined by the contact stiffness, \( k_c \), and the tangential stiffness, \( k_t \), which are derived from the Hertz–Mindlin model,\(^9\) and the modified gap stiffness, \( k_g \), by using the following equation:

\[
\mu_b = \frac{C_n(1-\beta)}{20\pi r} \left\{ \left( k_c + \frac{3}{2} k_t \right) + k_g \right\} = \mu_{bHM} + \mu_{bgf} \left\{ 1 - \frac{1}{\kappa a J_0(\kappa a)} \right\}, \tag{5}
\]

where \( \beta \) denotes the porosity; \( C_n \), the coordination number; \( r \), the grain radius; and \( \mu_{bHM} \) is a frequency-independent term (the Hertz–Mindlin term), \( \mu_{bgf} \) is the maximum gap stiffness term, and \( \mu_{bgf}(f) \) is a frequency-dependent term (gap stiffness term).

3. Shear wave measurements

3.1 Samples and method

Water-saturated silica sands with grain diameters of 0.193, 0.324, 0.545, and 0.917 mm were used as the samples. Each silica sand with single grain size was prepared by sieving. The measurement method is almost the same as that described in Ref. 4. The sample was placed in a cylindrical acrylic vessel with an inner diameter of 43 mm. The length of the vessel in different trials was 40, 80, 120, and 160 mm. The length could be changed by stacking 40 mm long pieces of tube in layers. The center frequency of the transmitter and the receiver is 500 kHz. The vertical stress on the sediment column was 17.6 kPa. Measurements were made at a temperature of 20°C using a constant temperature bath.

3.2 Results and discussions

The measured results of the shear wave speed dispersion and the frequency dependence of the attenuation coefficient are shown in Fig. 1. The 90% confidence intervals are
also provided in the figure. From Fig. 1(a), it is found that the shear wave speed increases with frequency at any grain sizes, except at higher frequencies for the largest grain size; in this case the shear wave speed decreases as the frequency increases. The shear wave speeds are greater at larger grain sizes than at smaller grain sizes. From Fig. 1(b), it is shown that the attenuation decreases as the grain size increases and the attenuation increases steeply at high frequencies for the grain sizes of 0.545 and 0.917 mm.

Fig. 1. Measured and calculated (a) shear wave speeds, $c_s$, and (b) attenuations, $x_s$, as a function of frequency, $f$, in four kinds of water-saturated silica sands with different grain sizes.

The dependence of the shear wave speed dispersion and the frequency dependence of the attenuation on the grain size are considered to be due to mainly the differences in the relaxation frequencies of the gap stiffness in the BIMGS model. This is because the aspect ratio in the BIMGS model is dependent on the grain size as shown in Ref. 4. The aspect ratio, $\alpha$, as a function of the grain size, $\phi = -\log_2 d$, $d$ is grain size in mm, is as follows: $^4$
\[
\alpha = (1.32 \times 10^{-4}) \times 10^{0.146\phi}.
\] (6)

Now, the measured results of the shear wave speed dispersion and the frequency dependence of the attenuation will be compared with the results calculated using the BIMGS model. The values of the model parameters used in the calculations are listed in Table 1. The grain diameter, \( d \), the grain density, \( \rho_r \), the porosity, \( \beta \), and the permeability, \( \kappa \), were measured. The values of the pore fluid parameters such as density, \( \rho_f \), the bulk modulus, \( K_r \), and the viscosity, \( \eta \), were literature values at temperatures of 20\(^\circ\)C. The value of the grain bulk modulus, \( K_r \), was also taken from literature. The value of the pore size, \( a_p \), was derived from the equation \( a_p = \left( d/3\right)\beta / (1 - \beta) \) \(^2\). The value of the structure factor, \( x_t \), was derived from the equation \( x_t = 1 + r(1 - \beta)/\beta \) \((r = 1/2)\). The aspect ratio, \( x \), was obtained from Eq. (6). The values of the Hertz–Mindlin shear modulus, \( \mu_{\text{HM}} \), and the maximum gap stiffness term of the frame shear modulus, \( \mu_{\text{HMC}} \), were inverted by fitting the measured shear wave speed dispersion to the calculated values. A heuristic correction factor for attenuation, \( C_a \), was taken to fit the measured values of the shear wave attenuation to the calculated values. \(^4\) The factor is defined as follows:

\[
k_g = k_{gr} + jC_au_{gr},
\] (7)

where \( k_{gr} \) and \( k_{sd} \) are the real and imaginary parts of the modified gap stiffness, \( k_{gr} \), respectively. The imaginary unit, \( j \), in Eq. (7) was dropped in Eq. (28) of Ref. 4. The BIMGS model configuration consists of a disk-like slit with radius, \( a \), and width, \( h \), as shown in Fig. 3(a) of Ref. 5. The disk-like slit in the model appear to be different from the real configuration. This may be one possible reason for the necessity to use the heuristic correction factor. The sands used in Ref. 4 and this study are different in grain size distribution and porosity. The former has grain size distribution (\( \phi \)-scale standard deviation is 0.41) with 0.368 in porosity, while the latter have uniform grain sizes with 0.378–0.389 in porosity. It is considered that the difference is one of the
causes of making a difference between both values of the factor. The above problems
remain to be solved in detail.

The measured and calculated shear wave speed dispersion and the frequency
dependence of the attenuation are shown in Fig. 1. The calculated shear wave speed
dispersion and attenuation is almost consistent with the measured results. However, it
is shown that there is a discrepancy between both results for small grain sizes at low
frequencies. The result remains to be identified. The larger issue is that there is nega-
tive shear wave speed dispersion for the largest grain size at higher frequencies in the meas-
ured results. Also, for the larger grain sizes of 0.545 and 0.917 mm, the measured
attenuations come to be greater than the calculated ones at high frequencies. These
phenomena are considered to be due to multiple scattering which are just the same as
compressional waves as shown in Ref. 12.

The speed dispersion is investigated by using the normalized frequency, \( \frac{f}{f_r} \),
instead of the frequency, \( f \), and the normalized speed, \( \frac{c_s}{c_{s0}} \) (\( c_{s0} \) is the calculated shear
wave speed at \( f = 0 \)) in order to make the relationship between the grain size depend-
ance of the shear wave speed dispersion and the aspect ratio in the BIMGS model
clear. The measured and calculated shear wave speeds as a function of the
normalized frequency are shown in Fig. 2(a). From Fig. 2(a), it can be seen that the
measured and calculated normalized speed dispersion for all grain sizes lies almost on
the same curve. Therefore, it is validated that Eq. (6) is applicable in the BIMGS
model. As pointed out before, it is shown that there is negative shear wave speed dis-
persion at higher \( \frac{f}{f_r} \) for the largest grain size.

The attenuation is also investigated by using the normalized frequency, \( \frac{f}{f_r} \),
and \( a_{sd} \) as the normalized attenuation. The attenuation due to the gap stiffness loss at
high frequencies, \( a_{\infty} \left( \frac{f}{f_r} \right) \), is approximately expressed as follows:

\[
\frac{a_{\infty} \left( \frac{f}{f_r} \right)}{f_r} \propto \frac{1}{d} \sqrt{\frac{f}{f_r}}. \tag{8}
\]

The equation above can be derived by using Eq. (26) in Ref. 4, and Eq. (6). This fact
is the reason that \( a_{sd} \) is taken as the normalized attenuation. The measured and calcu-
lated \( a_{sd} \) as a function of the normalized frequency, \( \frac{f}{f_r} \), are shown in Fig. 2(b). From
Fig. 2(b), it can be shown that the measured and calculated \( a_{sd} \) for all grain sizes lie
almost on the same curve except at higher \( \frac{f}{f_r} \). When the normalized frequency, \( \frac{f}{f_r} \),

![Fig. 2. (Color online) Measured and calculated (a) normalized shear wave speeds, \( \frac{c_s}{c_{s0}} \), and (b) \( a_{sd} \) as a function of the normalized frequency, \( \frac{f}{f_r} \), in four kinds of water-saturated silica sands with different grain sizes.](image-url)
exceeds approximately 2, the measured values for attenuation become greater than the calculated ones.

In order to examine the effects of shear wave multiple scattering in a similar way as those of compressional wave,\textsuperscript{12} the shear wave speed and attenuation are further investigated by using the dimensionless parameter, \( l_s d \) (where \( l_s = \omega \sigma \sigma \); wave number of shear wave), as the abscissa. The measured and calculated \( c_s / c_{s0} \) and \( \alpha_s d \) as a function of \( l_s d \) are shown in Fig. 3. There is no theoretical model for shear wave multiple scattering in granular marine sediments. However, some studies on multiple scattering for the media with solid or liquid inclusions in a solid matrix found that the frequency dependence of speed and attenuation for compressional and shear waves has almost the same trends.\textsuperscript{13–15} Then, assuming that the same forms as Eq. (22) in Ref. 16 and Eq. (35) in Ref. 12 can be applied for also shear waves, the following equations are obtained by fitting the measured shear wave speed dispersion and attenuation to the calculated values as follows:

\[
\frac{c_s}{c_{s0}} = \frac{c_s}{c_{s0}} \bigg|_{\text{no scattering}} - (3.00 \times 10^{-2})(l_s d)^{2.45}, \quad (9)
\]

\[
\alpha_s d = \alpha_s d \bigg|_{\text{no scattering}} + 0.16(l_s d)^4. \quad (10)
\]

The calculated results obtained by using Eqs. (9) and (10) are also shown in Fig. 3. From Fig. 3, it is seen that the measured results are consistent with the calculated results by using the BIMGS model in the range of \( l_s d \leq 0.5 \) and by using the BIMGS model plus multiple scattering effects in the range of \( l_s d \geq 0.5 \). These results are just the same as that in the case of compressional waves.\textsuperscript{12}

4. Conclusions

In this study, the grain-size dependence of shear wave speed dispersion and attenuation in four kinds of water-saturated silica sands with different grain sizes in the frequency range of 4–20 kHz were measured. The measured results were compared with those calculated using the BIMGS model. As a result, the grain size dependence of the aspect ratio in the BIMGS model could be validated. In addition, the effects of multiple scattering for larger grain sizes were demonstrated.
References and links